arXiv:hep-th/0012182v2 27 Dec 2000

An Exact Solution of BPS Junctions and Its Properties

Kenji Ito, Masashi Naganuma, Hodaka Oda* and Norisuke Sakai†

Department of Physics, Tokyo Institute of Technology,
Oh-okayama, Meguro, Tokyo 152-8551, Japan

We have obtained an exact solution for the BPS domain wall junction for a $\mathcal{N}=1$ supersymmetric theory in four dimensions and studied its properties. The model is a simplified version of the $\mathcal{N}=2$ $SU(2)$ gauge theory with $N_f=1$ broken to $\mathcal{N}=1$ by the mass of the adjoint chiral superfield. We define mode equations and demonstrate explicitly that fermion and boson with the same mass have to come in pairs except massless modes. We work out explicitly massless Nambu-Goldstone (NG) modes on the BPS domain wall junction. We find that their wave functions extend along the wall to infinity (not localized) and are not normalizable. It is argued that this feature is a generic phenomenon of NG modes on domain wall junctions in the bulk flat space in any dimensions. NG fermions exhibit a chiral structure in accordance with unitary representations of $(1,0)$ supersymmetry algebra where fermion and boson with the same mass come in pairs except massless modes which can appear singly. More detailed exposition of our results can be found in Refs.[1], [2].

1. Introduction

For sometime much attention has been paid to topological objects. Recently an interesting idea has been advocated to regard our world as a domain wall embedded in higher dimensional spacetime [3]. In this "brane world" scenario, our four-dimensional space-time on these topological objects is embedded in higher dimensional space-time. Most of the particles in the standard model should be realized as modes localized on the wall. Phenomenological implications of the idea have been extensively studied from many aspects. Another fascinating possibility has also been proposed to consider walls in the bulk spacetime with a negative cosmological constant[4].

Supersymmetry has been useful to achieve stability of solitonic solutions such as domain walls. Domain walls in supersymmetric theories can saturate the Bogomol'nyi bound and is called a BPS state[5]. It has also been noted that these BPS states possess a topological charge which becomes a central charge Z of the supersymmetry algebra [6],[7]. Thanks to the topological charge, these BPS states are guaranteed to be stable under arbitrary local fluctuations. The modes on the domain wall background have been worked out and are found to contain fermions and/or bosons localized on the wall in many cases[10],[11].

By interpolating two discrete degenerate vacua in separate regions of space, we obtain a domain wall[12],[13]. These walls are typically codimension one and breaks half of the supersymmetry which is called a 1/2 BPS state. If we have three or more discrete vacua in separate regions of space, segments of domain walls can meet at a one-dimensional junction and there arises a domain wall junction. The domain wall junction typically can preserve 1/4 of supersymmetry and is called a 1/4 BPS state.

In this report we summarize our recent results on the first exact solution of BPS domain wall junction in a supersymmetric gauge theory[1] and the detailed study of its properties[2].

2. BPS equations and $(1,0)$ SUSY

2.1. BPS equations

If the translational invariance is broken as is the case for domain walls and/or junctions, the

*The author gratefully acknowledges support from the Iwanami Fujukai Foundation.
†This work is supported in part by Grant-in-Aid for Scientific Research from the Japan Ministry of Education, Science and Culture for the Priority Area 291 and 707.

$\mathcal{N}=1$ superalgebra in general receives contributions from central charges [6],[7],[1],[2]. The anti-commutator between two left-handed supercharges has central charges Z_k, $k=1,2,3$

$$\{Q_\alpha, Q_\beta\} = 2i(\sigma^k\bar{\sigma}^0)_\alpha{}^\gamma \epsilon_{\gamma\beta} Z_k. \tag{1}$$

The four dimensional indices are denoted by Greek letters $\mu, \nu = 0,1,2,3$ instead of roman letters m, n. The anti-commutator between left- and right-handed supercharges receives a contribution from central charges Y_k, $k=1,2,3$ besides the energy-momentum four-vector P^μ, $\mu = 0, \cdots, 3$ of the system

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2(\sigma^\mu_{\alpha\dot\alpha} P_\mu + \sigma^k_{\alpha\dot\alpha} Y_k). \tag{2}$$

One may call Z_k and Y_k as $(1,0)$ and $(1/2,1/2)$ central charges in accordance with the transformation properties under the Lorentz group. Central charges, Z_k and Y_k, come from the total divergence, and they are non-vanishing when there are nontrivial differences in asymptotic behavior of fields in different region of spatial infinity as is the case of domain walls and junctions[8],[9]. Therefore these charges are topological in the sense that they are determined completely by the boundary conditions at infinity. For instance, we can take a general Wess-Zumino model with an arbitrary number of chiral superfields Φ^i, an arbitrary superpotential $\mathcal{W}$ and an arbitrary Kähler potential $K(\Phi^i, \Phi^{*j})$

$$\mathcal{L} = \int d^2\theta d^2\bar\theta K(\Phi^i, \Phi^{*j}) + \left[\int d^2\theta \mathcal{W}(\Phi^i) + \text{h.c.}\right], \tag{3}$$

and compute the anticommutators (1), (2) to find the central charges. The contributions to these central charges from bosonic components of chiral superfields are given by[9]

$$Z_k = 2\int d^3x\, \partial_k \mathcal{W}^*(A^*), \tag{4}$$

$$Y_k = i\epsilon^{knm} \int d^3x\, K_{ij^*} \partial_n (A^{*j} \partial_m A^i), \tag{5}$$

where $\epsilon^{123}=1$, and A^i is the scalar component of the i-th chiral superfield Φ^i and $K_{ij^*} = \partial^2 K(A^*, A)/\partial A^i \partial A^{*j}$ is the Kähler metric.

BPS domain wall is a 1/2 BPS state[7] and BPS domain wall junction is a 1/4 BPS state [8],[9]. To find the BPS equations satisfied by these BPS states, we consider a hermitian linear combination of operators Q and $\bar{Q}$ with an arbitrary complex two-vector β^α and its complex conjugate $\bar\beta^{\dot\alpha} = (\beta^\alpha)^*$ as coefficients

$$K = \beta^\alpha Q_\alpha + \bar\beta^{\dot\alpha} \bar{Q}_{\dot\alpha}. \tag{6}$$

We treat β^α as c-numbers rather than the Grassmann numbers. Since K is hermitian, the expectation value of the square of K over any state is non-negative definite

$$\langle S|K^2|S\rangle \geq 0. \tag{7}$$

The field configuration of static junction must be at least two-dimensional. If we assume, for simplicity, that it depends on x^1, x^2 then we obtain $\langle Z_3\rangle = \langle Y_1\rangle = \langle Y_2\rangle = 0$ from Eqs.(4) and (5), and the equality holds if and only if the linear combination of supercharges, K, is preserved by the state $|S\rangle$

$$K\,|S\rangle = 0. \tag{8}$$

In this case, the state $|S\rangle$ saturates the energy bound and is called a BPS state. We find that there are two candidates for the saturation of the energy bound[1];

$$H = H_{\rm I} \equiv |\langle -iZ_1 - Z_2\rangle| - \langle Y_3\rangle, \tag{9}$$

when $\bar\beta^{\dot 1} = \beta^1 \frac{\langle iZ_1+Z_2\rangle}{|\langle iZ_1+Z_2\rangle|}$, and $\beta^2 = \bar\beta^{\dot 2} = 0$,

$$H = H_{\rm II} \equiv |\langle iZ_1 - Z_2\rangle| + \langle Y_3\rangle, \tag{10}$$

when $\beta^1 = \bar\beta^{\dot 1} = 0$ and $\bar\beta^{\dot 2} = \beta^2 \frac{\langle -iZ_1+Z_2\rangle}{|\langle -iZ_1+Z_2\rangle|}$.

In the case of $H_{\rm I} \neq H_{\rm II}$, the BPS bound becomes $\langle H\rangle \geq \max\{H_{\rm I}, H_{\rm II}\}$. If $H_{\rm I} > H_{\rm II}$, then supersymmetry can only be preserved at $\langle H\rangle = H_{\rm I}$ and the only one combination of supercharges is conserved

$$\left(Q_1 + \frac{\langle iZ_1+Z_2\rangle}{|\langle iZ_1+Z_2\rangle|}\bar{Q}_{\dot 1}\right)|\langle H\rangle = H_{\rm I}\rangle = 0. \tag{11}$$

If $H_{\rm II} > H_{\rm I}$, then supersymmetry can only be preserved at $\langle H\rangle = H_{\rm II}$ and the only one combination of supercharges is conserved

$$\left(Q_2 + \frac{\langle -iZ_1+Z_2\rangle}{|\langle -iZ_1+Z_2\rangle|}\bar{Q}_{\dot 2}\right)|\langle H\rangle = H_{\rm II}\rangle = 0. \tag{12}$$

In the case of $H_{\rm I} = H_{\rm II}$, two candidates of BPS bounds coincide and BPS state conserves both of two supercharges, (11) and (12); this is a 1/2 BPS state.

For the general Wess-Zumino model in Eq.(3), the condition of supercharge conservation (11) for $H = H_{\rm I}$ applied to chiral superfield $\Phi^i = (A^i, \psi^i, F^i)$ gives after eliminating the auxiliary field F^i

$$2\frac{\partial A^i}{\partial \bar{z}} = -\Omega_+ F^i = \Omega_+ K^{-1ij^*} \frac{\partial \mathcal{W}^*}{\partial A^{*j}}, \tag{13}$$

where $\Omega_+ \equiv i\langle -iZ_1^* + Z_2^*\rangle/|\langle -iZ_1^* + Z_2^*\rangle|$, and complex coordinates $z = x^1 + ix^2, \bar{z} = x^1 - ix^2$, and the inverse of the Kähler metric K^{-1ij^*} are introduced. We can also consider gauge interactions where we should use covariant derivative instead of ordinary derivative. Moreover the same BPS condition (11) applied to vector superfield in the Wess-Zumino gauge $V = (v_\mu, \lambda, D)$ gives after eliminating the auxiliary field D

$$v_{12} = -D = \frac{1}{2}\sum_j A^{*j} e_j A^j, \tag{14}$$

and $v_{03} = 0, v_{01} = v_{31}, v_{23} = -v_{02}$, where $v_{\mu\nu} \equiv \partial_\mu v_\nu - \partial_\nu v_\mu$ and e_j is the charge of the field A^j. Here we assume for simplicity the minimal kinetic term both for the chiral superfield $K_{ij^*} = \delta_{ij^*}$ and for the vector superfield.

Similarly the condition of supercharge conservation (12) for $H = H_{\rm II}$ applied to chiral superfield in the Wess-Zumino model gives after eliminating the auxiliary field

$$2\frac{\partial A^i}{\partial z} = -\Omega_- F^i = \Omega_- K^{-1ij^*} \frac{\partial \mathcal{W}^*}{\partial A^{*j}}, \tag{15}$$

with $\Omega_- \equiv i\langle -iZ_1^* - Z_2^*\rangle/|\langle -iZ_1^* - Z_2^*\rangle|$. If $U(1)$ gauge interaction is present, the derivative $\partial A^i/\partial z$ should be replaced by the covariant derivative $\mathcal{D}_z A^i = \frac{1}{2}(\mathcal{D}_1 - i\mathcal{D}_2)A^i$. In this case the BPS condition applied to $U(1)$ vector superfield in the Wess-Zumino gauge becomes in the case of minimal kinetic terms

$$v_{12} = D = -\frac{1}{2}\sum_j A^{*j} e_j A^j, \tag{16}$$

and $v_{03} = 0, v_{01} = -v_{31}, v_{23} = v_{02}$.

2.2. The exact solution of BPS domain wall junction

In ref. [1], we have found an exact solution of BPS domain wall junction in a model motivated by the $\mathcal{N} = 2$ supersymmetric $SU(2)$ gauge theory with one flavor broken to $\mathcal{N} = 1$ by the mass of the adjoint chiral superfield. This model has the following chiral superfields with the charge assignment for the $U(1) \times U(1)'$ gauge group

$$\begin{array}{cccccccc} & \mathcal{M} & \tilde{\mathcal{M}} & \mathcal{D} & \tilde{\mathcal{D}} & \mathcal{Q} & \tilde{\mathcal{Q}} & T \\ U(1) & 0 & 0 & 1 & -1 & 1 & -1 & 0 \\ U(1)' & 1 & -1 & 1 & -1 & 0 & 0 & 0, \end{array} \tag{17}$$

interacting with a superpotential

$$\mathcal{W} = (T-\Lambda)\mathcal{M}\tilde{\mathcal{M}} + (T+\Lambda)\mathcal{D}\tilde{\mathcal{D}} + (T-m)\mathcal{Q}\tilde{\mathcal{Q}} - h^2 T,$$

where parameters Λ and h can be made real positive and a parameter m is complex[1]. In this model there are three discrete vacua,

$$\begin{aligned} &T = \Lambda,\ \mathcal{M} = \tilde{\mathcal{M}} = h,\ \mathcal{Q} = \tilde{\mathcal{Q}} = \mathcal{D} = \tilde{\mathcal{D}} = 0, \\ &T = m,\ \mathcal{Q} = \tilde{\mathcal{Q}} = h,\ \mathcal{M} = \tilde{\mathcal{M}} = \mathcal{D} = \tilde{\mathcal{D}} = 0, \\ &T = -\Lambda,\ \mathcal{D} = \tilde{\mathcal{D}} = h,\ \mathcal{Q} = \tilde{\mathcal{Q}} = \mathcal{M} = \tilde{\mathcal{M}} = 0, \end{aligned}$$

which are called vacuum 1, 2, and 3 with $\mathcal{W}_1 = -h^2\Lambda$, $\mathcal{W}_2 = -h^2 m$ and $\mathcal{W}_3 = h^2\Lambda$ respectively. When $m = i\sqrt{3}\Lambda$, this model becomes Z_3 symmetric. Thus three half walls are expected to connect at the junction with relative angles of $2\pi/3$. For definiteness, we specify the boundary condition where the wall 1 extends along the negative x^2 axis separating the vacuum 1 ($x^1 > 0$) and 3 ($x^1 < 0$). If we have only the wall 1, we obtain the central charge Z_k (vanishing Y_k) and find the two conserved supercharges from Eqs.(11) and (12) as

$$Q^{(1)} = \frac{1}{\sqrt{2}}(\mathrm{e}^{-i\frac{\pi}{4}}Q_2 + \mathrm{e}^{i\frac{\pi}{4}}\overline{Q}_{\dot{2}}), \tag{18}$$

$$Q^{(2)} = \frac{1}{\sqrt{2}}(\mathrm{e}^{i\frac{\pi}{4}}Q_1 + \mathrm{e}^{-i\frac{\pi}{4}}\overline{Q}_{\dot{1}}). \tag{19}$$

The other two walls have also two conserved supercharges : at wall 2,

$$Q^{(3)} = \frac{1}{\sqrt{2}}(\mathrm{e}^{-i\frac{\pi}{12}}Q_1 + \mathrm{e}^{i\frac{\pi}{12}}\overline{Q}_{\dot{1}}) \tag{20}$$

besides $Q^{(1)}$, and at wall 3,

$$Q^{(4)} = \frac{1}{\sqrt{2}}(\mathrm{e}^{-i\frac{5\pi}{12}}Q_1 + \mathrm{e}^{i\frac{5\pi}{12}}\overline{Q}_{\dot{1}}), \tag{21}$$

besides $Q^{(1)}$. When these three half walls coexist, we can have only one common conserved supercharge $Q^{(1)}$. In fact we find that the domain wall junction configuration conserves precisely this single combination of supercharges, even though it has also another central charge Y_k contributing. Correspondingly we obtain the BPS equations (15) and (16) for $H = H_{\rm II}$ with $\Omega_- = -1$. The BPS equations (16) for the vector superfield can be trivially satisfied by $v_\mu = 0$ and $D = 0$. The BPS equations (15) for chiral superfields become in this case

$$2\frac{\partial A^i}{\partial z} = -\frac{\partial \mathcal{W}^*}{\partial A^{*i}}, \tag{22}$$

assuming the minimal kinetic term. The solution for these BPS equations is given by[1],

$$\begin{aligned}
\mathcal{M}(z,\bar{z}) &= \tilde{\mathcal{M}}(z,\bar{z}) = \frac{\sqrt{2}\Lambda s}{s+t+u}, \\
\mathcal{D}(z,\bar{z}) &= \tilde{\mathcal{D}}(z,\bar{z}) = \frac{\sqrt{2}\Lambda t}{s+t+u}, \\
\mathcal{Q}(z,\bar{z}) &= \tilde{\mathcal{Q}}(z,\bar{z}) = \frac{\sqrt{2}\Lambda u}{s+t+u}, \qquad (23)\\
T(z,\bar{z}) &= \frac{2\Lambda}{\sqrt{3}}\frac{e^{-i\frac{1}{6}\pi}s + e^{-i\frac{5}{6}\pi}t + e^{i\frac{1}{2}\pi}u}{s+t+u} + \frac{i}{\sqrt{3}}\Lambda,
\end{aligned}$$

where $s = \exp\left[\frac{2}{\sqrt{3}}\Lambda \mathrm{Re}\left(e^{i\frac{1}{6}\pi}z\right)\right]$, $t = \exp\left[\frac{2}{\sqrt{3}}\Lambda \mathrm{Re}\left(e^{i\frac{5}{6}\pi}z\right)\right]$, and $u = \exp\left[\frac{2}{\sqrt{3}}\Lambda \mathrm{Re}\left(e^{-i\frac{1}{2}\pi}z\right)\right]$.

This model is motivated by the softly broken $\mathcal{N} = 2$ $SU(2)$ gauge theory with one flavor. However, we can simplify the model without spoiling the solvability to obtain a Wess-Zumino model consisting of purely chiral superfields by the following procedure. The vector superfields actually serve to constrain chiral superfields to have the identical magnitude pairwise through $D = 0$ to satisfy the BPS equation (16) for vector superfields: $|\tilde{\mathcal{M}}| = |\mathcal{M}|, |\tilde{\mathcal{D}}| = |\mathcal{D}|, |\tilde{\mathcal{Q}}| = |\mathcal{Q}|$. Therefore we can eliminate the vector superfields and reduce the number of chiral superfields by identifying pairwise $\tilde{\mathcal{M}} = \mathcal{M}, \tilde{\mathcal{D}} = \mathcal{D}, \tilde{\mathcal{Q}} = \mathcal{Q}$ [2],[14].

2.3. Unitary representations of $(1,0)$ supersymmetry algebra

Let us examine states on the background of a domain wall junction from the point of view of surviving symmetry. In the case of the BPS states satisfying the BPS equation (15) corresponding to $H = H_{\rm II}$, we have only one surviving supersymmetry charge $Q^{(1)}$, two translation generators H, P^3, and one Lorentz generator J^{03}, out of the $\mathcal{N} = 1$ four dimensional super Poincaré generators. Since we are interested in excitation modes on the background of the domain wall junction, we define the hamiltonian $H' = H - \langle H\rangle$ measured from the energy $\langle H\rangle$ of the background configuration. By projecting from the supersymmetry algebra (1), (2) with central charges in four dimensions, we immediately find

$$\left(Q^{(1)}\right)^2 = H' - P^3. \tag{24}$$

We also obtain the Poincaré algebra in $1+1$ dimensions

$$[J^{03}, Q^{(1)}] = \frac{i}{2}Q^{(1)},\ [J^{03}, H' \mp P^3] = i(H' \mp P^3).$$

Other commutation relations vanishes trivially. Together they form the $(1,0)$ supersymmetry algebra on the domain wall junction as anticipated[8].

To obtain unitary representations, we can diagonalize H' and P^3

$$H'|E,p^3\rangle = E|E,p^3\rangle,\ P^3|E,p^3\rangle = p^3|E,p^3\rangle, \tag{25}$$

with $E \geq |p^3|$ and combine them by means of $Q^{(1)}$. If $E - p^3 > 0$, we can construct bosonic state from fermionic state and vice versa by operating $Q^{(1)}$ on the states $(|B\rangle, |F\rangle)$, $|B\rangle = Q^{(1)}|F\rangle/\sqrt{E-p^3}$ and $|F\rangle = Q^{(1)}|B\rangle/\sqrt{E-p^3}$, obtaining a doublet representation. If $E - p^3 = 0$, operating by $Q^{(1)}$ on the state gives an unphysical zero norm state

$$\left|Q^{(1)}|E,p^3\rangle\right|^2 = \langle E,p^3|H' - P^3|E,p^3\rangle = 0. \tag{26}$$

Then the massless right-moving state $|E, p^3 = E\rangle$ is a singlet representation. This singlet state can either be boson or fermion. Thus we find that there are only two types of representations of the $(1,0)$ supersymmetry algebra, doublet and singlet. We also find that massive modes should appear in pairs of boson and fermion, whereas the massless right-moving mode can appear singly without accompanying a state with

opposite statistics. This provides an interesting possibility of a chiral structure for fermions.

If another BPS equation (13) corresponding to $H = H_{\rm I}$ is satisfied instead of Eq. (15), we have $(0,1)$ supersymmetry and the left-moving massless states can appear as singlets.

3. Nambu-Goldstone and other modes on the junction

3.1. Mode equation on the junction

Since the vector superfields have no nontrivial field configurations, Nambu-Goldstone modes have no component of vector superfield. Moreover we can replace our model, if we wish, by another model with purely chiral superfields without spoiling the essential features including the solvability. Consequently we shall neglect vector superfields and consider the general Wess-Zumino model in Eq.(3) in the following. For simplicity we assume the minimal kinetic term here $K_{ij^*} = \delta_{ij^*}$.

Let us consider quantum fluctuations A'^i, ψ^i around a classical solution $A^i_{\rm cl}$ which satisfies the BPS equations (13) and (14) for $H = H_{\rm I}$ or (15) and (16) for $H = H_{\rm II}$. The linearized equation for fermions is given by

$$- i\bar\sigma^\mu\partial_\mu\psi^i - \frac{\partial^2\mathcal{W}^*}{\partial A^{*i}_{\rm cl}\partial A^{*j}_{\rm cl}}\bar\psi^j = 0 \tag{27}$$

$$- i\sigma^\mu\partial_\mu\bar\psi^i - \frac{\partial^2\mathcal{W}}{\partial A^{i}_{\rm cl}\partial A^{j}_{\rm cl}}\psi^j = 0. \tag{28}$$

To separate variables for fermion equations, it is more convenient to use a gamma matrix representation where direct product structure of 2×2 matrices for (x^0, x^3) and (x^1, x^2) space is manifest. Transforming from such a representation to the Weyl representation which we are using, we can define the fermionic modes $\psi^i_{n\alpha}, \bar\psi^{i\dot\beta}_n$ combining components of left-handed and right-handed spinors by means of the following operators

$$\mathcal{O}_1{}^i{}_j \equiv \begin{bmatrix} -\frac{\partial^2\mathcal{W}^*}{\partial A^{*i}_{\rm cl}\partial A^{*j}_{\rm cl}} & -i\left(-\partial_1 + i\partial_2\right)\delta^i_j \\ -i\left(\partial_1 + i\partial_2\right)\delta^i_j & -\frac{\partial^2\mathcal{W}}{\partial A^{i}_{\rm cl}\partial A^{j}_{\rm cl}} \end{bmatrix} \tag{29}$$

$$\mathcal{O}_2{}^i{}_j \equiv \begin{bmatrix} -\frac{\partial^2\mathcal{W}}{\partial A^{i}_{\rm cl}\partial A^{j}_{\rm cl}} & -i\left(\partial_1 - i\partial_2\right)\delta^i_j \\ -i\left(-\partial_1 - i\partial_2\right)\delta^i_j & -\frac{\partial^2\mathcal{W}^*}{\partial A^{*i}_{\rm cl}\partial A^{*j}_{\rm cl}} \end{bmatrix} \tag{30}$$

$$\mathcal{O}_1{}^i{}_j \begin{bmatrix} \bar\psi^{j\dot 1}_n \\ \psi^j_{n2} \end{bmatrix} = -im^{(1)}_n \begin{bmatrix} \psi^i_{n1} \\ \bar\psi^{i\dot 2}_n \end{bmatrix} \tag{31}$$

$$\mathcal{O}_2{}^i{}_j \begin{bmatrix} \psi^j_{n1} \\ \bar\psi^{j\dot 2}_n \end{bmatrix} = im^{(2)}_n \begin{bmatrix} \bar\psi^{i\dot 1}_n \\ \psi^i_{n2} \end{bmatrix}, \tag{32}$$

where the mass eigenvalues $m^{(1)}_n, m^{(2)}_n$ are real. Please note a peculiar combination of left- and right-handed spinor components to define eigenfunctions. We can expand ψ^i in terms of these mode functions

$$\psi^i_\alpha(x^0,x^1,x^2,x^3) = \sum_n \begin{pmatrix} b_n(x^0,x^3)\psi^i_{n1}(x^1,x^2) \\ c_n(x^0,x^3)\psi^i_{n2}(x^1,x^2) \end{pmatrix} \tag{33}$$

Since $\psi(x^0,x^1,x^2,x^3)$ is a Majorana spinor, the coefficient fermionic fields b_n, c_n are real. The linearized equations (27) (28) for the fermion gives a Dirac equation in $1+1$ dimensions for the coefficient fermionic fields $\phi_n = (c_n, ib_n)$ with two mass parameters $m^{(1)}_n, m^{(2)}_n$

$$\left[-i\gamma^a\partial_a - m^{(1)}_n\frac{1+\rho_3}{2} - m^{(2)}_n\frac{1-\rho_3}{2}\right]\phi_n = 0, \tag{34}$$

where we use Pauli matrices $\rho_a,\ a = 1,2,3$ to construct the 2×2 gamma matrices in $1+1$ dimensions $(\gamma^0,\gamma^1) = (\rho_1, i\rho_2)$. Since we have a Majorana spinor in $1+1$ dimensions which does not allow chiral rotations, we have two distinct real mass parameters $m^{(1)}_n, m^{(2)}_n$.

Similarly we retain the part of the Lagrangian quadratic in fluctuations and eliminate the auxiliary fields F^i to obtain the linearized equation for the scalar fluctuations, $A'^i = A^i - A^i_{\rm cl}$,

$$\begin{aligned} &- \partial_\mu\partial^\mu A'^{*i} + \frac{\partial^2\mathcal{W}}{\partial A^i_{\rm cl}\partial A^k_{\rm cl}}\frac{\partial^2\mathcal{W}^*}{\partial A^{*k}_{\rm cl}\partial A^{*j}_{\rm cl}}A'^{*j} \\ &+ \frac{\partial^3\mathcal{W}}{\partial A^i_{\rm cl}\partial A^k_{\rm cl}\partial A^j_{\rm cl}}\frac{\partial\mathcal{W}^*}{\partial A^{*k}_{\rm cl}}A'^j = 0. \end{aligned} \tag{35}$$

In order to separate variables in x^0, x^3 and x^1, x^2 we have to define mode equations on the background which has a nontrivial dependence in two dimensions, x^1, x^2. The bosonic modes $A'^i_n(x^1,x^2)$ can easily be defined in terms of a differential operator $\mathcal{O}_B$ in x^1, x^2 space

$$\mathcal{O}_B{}^i{}_j \begin{bmatrix} A'^{*j}_n \\ A'^j_n \end{bmatrix} = M^2_n \begin{bmatrix} A'^{*i}_n \\ A'^i_n \end{bmatrix}, \tag{36}$$

where the eigenvalue M_n^2 has to be real from Majorana condition. The quantum fluctuation for scalar can be expanded in terms of these mode functions to obtain a real scalar field equation with the mass M_n for the coefficient bosonic field $a_n(x^0, x^3)$,

$$A'^i(x^0, x^1, x^2, x^3) = \sum_n a_n(x^0, x^3) A'^i_n(x^1, x^2) \quad (37)$$

$$\left(\partial_0^2 - \partial_3^2 + M_n^2\right) a_n(x^0, x^3) = 0. \quad (38)$$

To relate the mass eigenvalues of fermions and bosons, let us multiply two differential operators for fermions $\mathcal{O}_2$ to $\mathcal{O}_1$. In this ordering, we can use the BPS equation (15) corresponding to $H = H_{\rm II}$ to find the differential operator for bosons $\mathcal{O}_B$

$$\mathcal{O}^i_{2k}\mathcal{O}^k_{1j} = U\mathcal{O}^i_{Bj}U^{-1}, \quad (39)$$

$$U = \begin{bmatrix} e^{i\frac{\pi}{4}}\Omega_-^{\frac{1}{2}} & 0 \\ 0 & e^{-i\frac{\pi}{4}}\Omega_-^{-\frac{1}{2}} \end{bmatrix}. \quad (40)$$

Therefore the BPS equation (15) corresponding to $H = H_{\rm II}$ guarantees that the existence of a solution $\bar\psi_n^{i\dot 1}, \psi^i_{n2}$ of fermionic mode equations implies the existence of a solution of bosonic mode equations with the mass squared $M_n^2 = m_n^{(1)} m_n^{(2)}$

$$A'^{*i}_n = e^{-i\frac{\pi}{4}}\Omega_-^{-\frac{1}{2}}\bar\psi_n^{i\dot 1}, \qquad A'^i_n = e^{i\frac{\pi}{4}}\Omega_-^{\frac{1}{2}}\psi^i_{n2}. \quad (41)$$

If another BPS equation (13) corresponding to $H = H_{\rm I}$ is valid, operator multiplication with different ordering gives the same bosonic operator whose rows and columns are interchanged

$$\mathcal{O}^i_{1k}\mathcal{O}^k_{2j} = U'\mathcal{O}^i_{Bj}U'^{-1}, \quad (42)$$

$$U' = \begin{bmatrix} 0 & e^{i\frac{\pi}{4}}\Omega_+^{-\frac{1}{2}} \\ -e^{-i\frac{\pi}{4}}\Omega_+^{\frac{1}{2}} & 0 \end{bmatrix}. \quad (43)$$

Therefore the BPS equation (13) corresponding to $H = H_{\rm I}$ guarantees that the existence of a solution $\bar\psi_n^{i\dot 2}, \psi^i_{n1}$ of fermionic mode equations implies the existence of a solution of bosonic mode equations with the mass squared $M_n^2 = m_n^{(1)} m_n^{(2)}$

$$A'^{*i}_n = -e^{i\frac{\pi}{4}}\Omega_+^{-\frac{1}{2}}\bar\psi_n^{i\dot 2}, \qquad A'^i_n = e^{-i\frac{\pi}{4}}\Omega_+^{\frac{1}{2}}\psi^i_{n1}. \quad (44)$$

Therefore we find that all massive states come in pairs of boson and fermion with the same mass squared $M_n^2 = m_n^{(1)} m_n^{(2)}$ in accordance with the result of the unitary representation of the $(1,0)$ supersymmetry algebra.

3.2. Nambu-Goldstone modes

Since we are usually most interested in a low energy effective field theory, we wish to study massless modes here. If global continuous symmetries are broken spontaneously, there occur associated massless modes which are called the Nambu-Goldstone modes. To find the wave functions of the Nambu-Goldstone modes, we perform the associated global transformations and evaluate the transformed configuration by substituting the classical field. For supersymmetry we obtain nontrivial wave function by substituting the classical field $A^i_{\rm cl}(x^1, x^2)$ and $F^i_{\rm cl}(x^1, x^2)$ to the transformation of fermions by a Grassmann parameter ξ, since classical field configuration of fermion vanishes $\psi^i_{\rm cl} = 0$

$$\delta_\xi\psi^i = i\sqrt{2}\sigma^\mu\bar\xi\partial_\mu A^i_{\rm cl} + \sqrt{2}\xi F^i_{\rm cl}. \quad (45)$$

If the BPS equation (15) for the junction background is valid, we obtain

$$\delta_\xi\psi^i = \quad (46)$$
$$\sqrt{2}\left[(i\sigma^1\bar\xi - \Omega_-^*\xi)\partial_1 A^i_{\rm cl} + (i\sigma^2\bar\xi + i\Omega_-^*\xi)\partial_2 A^i_{\rm cl}\right]$$

We see that there is one conserved direction in the Grassmann parameter:

$$i\sigma^1\bar\xi = \Omega_-^*\xi \text{ and } \sigma^2\bar\xi = -\Omega_-^*\xi. \quad (47)$$

The other three real Grassmann parameters ξ correspond to broken supercharges. For our exact solution, for instance, we find it convenient to choose the three broken supercharges as the following real supercharges

$$Q_{\rm I} = \frac{1}{\sqrt{2}}(e^{i\pi/4}Q_2 + e^{-i\pi/4}\bar Q_{\dot 2}), \quad (48)$$

$$Q_{\rm II} = \frac{1}{\sqrt{2}}(e^{-i\pi/4}Q_1 + e^{i\pi/4}\bar Q_{\dot 1}), \quad (49)$$

$$Q_{\rm III} = \frac{1}{\sqrt{2}}(e^{i\pi/4}Q_1 + e^{-i\pi/4}\bar Q_{\dot 1}). \quad (50)$$

Then the corresponding massless mode functions are given by

$$\psi_0^{({\rm I})i}(x^1, x^2) = \begin{pmatrix} 4\partial_z A^i_{\rm cl}(x^1, x^2)e^{-i\pi/4} \\ 0 \end{pmatrix}, \quad (51)$$

$$\psi_0^{({\rm II})i}(x^1, x^2) = \begin{pmatrix} 0 \\ 2\partial_1 A^i_{\rm cl}(x^1, x^2)e^{i\pi/4} \end{pmatrix}, \quad (52)$$

$$\psi_0^{({\rm III})i}(x^1, x^2) = \begin{pmatrix} 0 \\ 2\partial_2 A^i_{\rm cl}(x^1, x^2)e^{i\pi/4} \end{pmatrix}. \quad (53)$$

Since the transformation parameter should correspond to the Nambu-Goldstone field with zero momentum and energy, the three transformation parameters ξ should be promoted to three real fermionic fields in x^0, x^3 space, $b_0^{(\mathrm{I})}(x^0, x^3), c_0^{(\mathrm{II})}(x^0, x^3)$, and $c_0^{(\mathrm{III})}(x^0, x^3)$, to obtain the Nambu-Goldstone component of the mode expansion

$$\begin{aligned}
&\psi^i(x^0,x^1,x^2,x^3) = b_0^{(\mathrm{I})}(x^0,x^3)\psi_0^{(\mathrm{I})i}(x^1,x^2) \\
&+c_0^{(\mathrm{II})}(x^0,x^3)\psi_0^{(\mathrm{II})i}(x^1,x^2) \\
&+c_0^{(\mathrm{III})}(x^0,x^3)\psi_0^{(\mathrm{III})i}(x^1,x^2) \\
&+\sum_{n>0}\begin{pmatrix} b_n(x^0,x^3)\psi_{n1}^i(x^1,x^2) \\ c_n(x^0,x^3)\psi_{n2}^i(x^1,x^2)\end{pmatrix}.
\end{aligned} \tag{54}$$

We have explicitly displayed three massless Nambu-Goldstone fermion components distinguishing from the massive ones ($n > 0$). The Dirac equation for the coefficient fermionic fields (34) shows that $b_0^{(\mathrm{I})}(x^0 - x^3)$ is a right-moving massless mode, and $c_0^{(\mathrm{II})}(x^0 + x^3)$, and $c_0^{(\mathrm{III})}(x^0 + x^3)$ are left-moving modes.

We plot the absolute values of $|\psi_0^{(a)i=T}|$ of the $i = T$ component of the wave function of the Nambu-Goldstone fermions $a = \mathrm{I}, \mathrm{II}, \mathrm{III}$ in Fig. 1. We can see that Nambu-Goldstone fermions have wave functions which extend to infinity along three walls. They become identical to fermion zero modes on at least two of the walls asymptotically and hence they are not localized around the center of the junction. We can construct a linear combination of the Nambu-Goldstone fermions to have no support along one out of the three walls. However, no linear combination of these Nambu-Goldstone fermions can be formed which does not have support extended along any of the wall. Therefore these wave functions are not localized and are not normalizable. This fact means that the low energy dynamics of BPS junction cannot be described by a $1+1$ dimensional effective field theory with a discrete particle spectrum.

Similarly the Nambu-Goldstone bosons corresponding to the broken translation $P^a, a = 1, 2$ are given by

$$A_0^{(a)}(x^1, x^2) = \partial_a A_{\mathrm{cl}}^i(x^1, x^2), \qquad a = 1, 2. \tag{55}$$

These two bosonic massless modes consist of two left-moving modes and two right-moving modes. On the other hand, we have seen already that there are two left-moving massless Nambu-Goldstone fermions and one right-moving massless Nambu-Goldstone fermion. These two left-moving Nambu-Goldstone bosons and fermions form two doublets of the $(1, 0)$ supersymmetry algebra. The right-moving modes are asymmetric in bosons and fermions: two Nambu-Goldstone bosons and a single Nambu-Goldstone fermion. These three states are all singlets of the $(1, 0)$ supersymmetry algebra in accordance with our analysis in sect.2.3. Therefore we obtained a chiral structure of Nambu-Goldstone fermions on the junction background configuration.

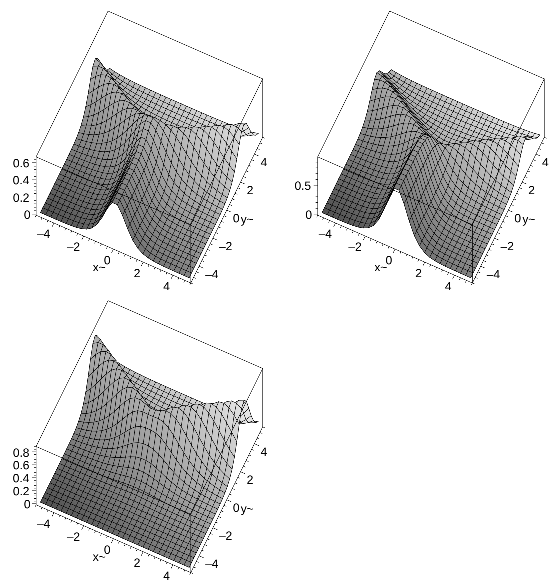

Figure 1. The bird's eye view of the absolute value of the $i = T$ component of the wave functions of the Nambu-Goldstone fermions on the junction in the (x^1, x^2) space

3.3. Non-normalizability of the Nambu-Goldstone fermions

We would like to argue that our observation is a generic feature of the Nambu-Goldstone fermions on the domain wall junction in a flat space in the bulk: Nambu-Goldstone fermions are not localized at the junction and hence are not normalizable, if they are associated with the supersymmetry breaking due to the coexistence of nonparallel domain walls. The following observation is behind this assertion. A single domain wall breaks only a half of supercharges. Nonparallel wall also breaks half of supercharges, some of which may be linear combinations of the supercharges already broken by the first wall. If the junction configuration is a 1/4 BPS state, linearly independent ones among these two sets of broken supercharges of nonparallel walls become $\frac{3}{4}$ of the original supercharges.

To see in more detail, let us first note that the junction configuration reduces asymptotically to a wall if one goes along the wall, say the wall 1. Let us denote the number of original supercharges to be N. On the first wall, a half of the original supersymmetry $(Q^{(1)}, \cdots, Q^{(N)})$ is broken. We call these broken supercharges as $Q^{(1)}, \cdots, Q^{(N/2)}$. Consequently we have Nambu-Goldstone fermions localized around the core of the wall and is constant along the wall. In the junction configuration, we have other walls which are not parallel to the first wall. Asymptotically far away along one of such walls, say wall 2, another half of the supersymmetry $Q'^{(1)}, \cdots, Q'^{(N/2)}$ is broken. If the junction is a 1/4 BPS state, a half of these, say $Q'^{(1)}, \cdots, Q'^{(N/4)}$, is a linear combination of $Q^{(1)}, \cdots, Q^{(N/2)}$ broken already on the wall 1. The other half, $Q'^{(\frac{N}{4}+1)}, \cdots, Q'^{(\frac{N}{2})}$ are unbroken on the wall 1. Altogether a quarter of the original supercharges remain unbroken. Consequently the Nambu-Goldstone fermions corresponding to $Q'^{(1)}, \cdots, Q'^{(N/4)}$ have a wave function which extends to infinity and approaches a constant profile along both the walls 1 and 2. Those modes corresponding to $Q'^{(\frac{N}{4}+1)}, \cdots, Q'^{(\frac{N}{2})}$ have support only along the wall 2, and those corresponding to the linear combinations of $Q^{(1)}, \cdots, Q^{(N/2)}$ orthogonal to $Q'^{(1)}, \cdots, Q'^{(N/4)}$ have support only along the wall 1. Thus we find that any linear combinations of the Nambu-Goldstone fermions have to be infinitely extended along at least one of the walls which form the junction configuration. Therefore the Nambu-Goldstone fermions associated with the coexistence of nonparallel domain walls are not localized at the junction and are not normalizable.

In our exact solution, domain wall junction configuration reduces asymptotically to the wall 1 at $x^2 \to -\infty$ with fixed x^1. On the wall, only two supercharges in Eqs.(48)– (50) are broken

$$Q_{\rm I} = \frac{1}{\sqrt{2}}(e^{i\pi/4}Q_2 + e^{-i\pi/4}\overline{Q}_{\dot{2}}), \tag{56}$$

$$Q_{\rm II} = \frac{1}{\sqrt{2}}(e^{-i\pi/4}Q_1 + e^{i\pi/4}\overline{Q}_{\dot{1}}), \tag{57}$$

and there are two corresponding Nambu-Goldstone fermions which become domain wall zero modes asymptotically

$$\begin{aligned} \psi_0^{({\rm I})i}(x^1,x^2) &= \begin{pmatrix} 4\partial_z A^i_{\rm cl}(x^1,x^2)e^{-i\pi/4} \\ 0 \end{pmatrix} \quad (58)\\ &\to \begin{pmatrix} 2\partial_1 A^{i\rm wall}_{\rm cl}(x^1)e^{-i\pi/4} \\ 0 \end{pmatrix}, \\ \psi_0^{({\rm II})i}(x^1,x^2) &= \begin{pmatrix} 0 \\ 2\partial_1 A^i_{\rm cl}(x^1,x^2)e^{i\pi/4} \end{pmatrix} \\ &\to \begin{pmatrix} 0 \\ 2\partial_1 A^{i\rm wall}_{\rm cl}(x^1)e^{i\pi/4} \end{pmatrix}. \quad (59) \end{aligned}$$

These wave functions are localized on the core of the wall 1 in the x^1 direction and are constant along the wall. Along the other walls we find two broken supercharges one of which is identical to one of the broken supercharges, $Q_{\rm I}$. The other broken supercharge is $Q'_{\rm II}$ on the wall 2 and $Q''_{\rm II}$ on the wall 3. There are only two independent supercharges among $Q_{\rm II}$, $Q'_{\rm II}$, and $Q''_{\rm II}$. Together with $Q_{\rm I}$ we obtain three independent broken supercharges. We can construct a linear combination of the Nambu-Goldstone fermions to have no support along one out of the three walls. However, any linear combination has nonvanishing wave function which becomes fermion zero mode on at least one of the wall asymptotically. Therefore the associated Nambu-Goldstone fermions have

support which is infinitely extended at least along two of the walls.

If a single wall is present, we can explicitly construct a plane wave solution propagating along the wall, which may be called a spin wave and is among massive modes on the wall background. Even if there are several walls forming a junction configuration, we can consider excitation modes which reduce to the spin wave modes along each wall. They should be a massive mode on the domain wall junction background. The Nambu-Goldstone mode on the domain wall junction is the zero wave number limit of such a spin wave mode. This physical consideration suggests that the massless Nambu-Goldstone fermion is precisely the vanishing wave number (along the wall) limit of the massive spin wave mode.

Let us note that our argument does not apply to models with the bulk cosmological constant. In such models, massless graviton is localized on the background of intersection of walls[15]. In that case, massless mode is a distinct mode different from the massless limit of the massive continuum, although the massless mode is buried at the tip of the continuum of massive modes. The normalizability of the massless graviton is guaranteed by the Anti de Sitter geometry away from the junction or intersection including the direction along the wall.

3.4. Negative contribution of central charge Y_3 to junction mass

Next we discuss the sign of the contribution of the central charge Y_3 to the mass of the junction configuration. We can use the Stokes theorem to obtain an expression for the central charge Y_3 as a contour integral[1],[9]

$$Y_3 = \int dx^3\, i \int d^2x \left[\partial_1\left(K_i\partial_2 A^i\right) - \partial_2\left(K_i\partial_1 A^i\right)\right] = \int dx^3\, i \oint K_i dA^i, \tag{60}$$

where $K_i \equiv \partial K/\partial A^i$ is a derivative of the Kähler potential K. This contour integral in the field space should be done as a map from a counterclockwise contour in the infinity of $z = x^1 + ix^2$ plane. Only complex fields can contribute to Y_3. Let us assume for simplicity that there is only one field which can contribute to Y_3 as in our exact solution.

Eq.(60) shows that the central charge Y_3 becomes negative (positive), if the asymptotic counterclockwise contour in x^1, x^2 is mapped into a counterclockwise (clockwise) contour in field space. On the other hand, the sign of the contribution of the central charge Y_3 to the mass of the junction configurtion is determined by the formula, $H = H_{\rm II} = |\langle iZ_1 - Z_2\rangle| + \langle Y_3\rangle$ or $H = H_{\rm I} = |\langle -iZ_1 - Z_2\rangle| - \langle Y_3\rangle$. The choice of these mass formulas are in turn deterined by the map of the asymptotic counterclockwise contour in x^1, x^2 space to a counterclockwise or clockwise contour in the superpotential space $\mathcal{W}$. Combining these two observations, we conclude that the contribution of the central charge Y_3 to the mass of the junction configuration is negative if the sign of rotations is the same in field space A^i and in superpotential space $\mathcal{W}$, and positive if the sign of rotations is opposite.

The field configuration moves counterclockwise in field space in our exact solution in (23) and then the central charge is negative in this solution. Since the exact solution satisfies the BPS equation for the case $H = H_{\rm II}$, the central charge contributes to the mass of the junction configuration negatively. Therefore we should not consider the central charge Y_3 alone as the physical mass of the junction at the center. In the junction configuration, the junction at the center cannot be separated from the walls. We also can find a solution for the other case of $H = H_{\rm I}$ in our model. The solution is just a configuration obtained by a reflection $x^1 \to -x^1$. Then the central charge is positive, but the contribution to the mass $H = H_{\rm I}$ becomes again negative. In either solution, the rotation in field T space has the same sign as the rotation in superpotential $\mathcal{W}$ space. Therefore central charge Y_3 contributes negatively to the mass of the junction, irrespective of the choice of $H = H_{\rm I}$ or $H = H_{\rm II}$.

More recently this feature of negative contribution of Y_3 to the junction mass is studied from a different viewpoint and it is argued that this feature is valid in most situations except possibly in contrived models[14].

4. Summary

1. We have obtained an exact solution of domain wall junction in a four-dimensional $\mathcal{N} = 1$ SUSY $U(1) \times U(1)'$ gauge theory. The model has three pairs of chiral superfields and is motivated by the $\mathcal{N} = 2$ $SU(2)$ gauge theory with one flavor perturbed by an adjoint scalar mass.

2. Mode functions are defined and are shown to appear in a boson-fermion pair of identical mass for massive modes.

3. Nambu-Goldstone fermions exhibit a chiral structure in accordance with $(1, 0)$ supersymmetry.

4. Nambu-Goldstone fermions are not normalizable. The domain wall junction configuration preserves only $1/4$ of the original supercharges, but modes on the junction does not reduce to a $1 + 1$ dimensional field theory with a discrete mass spectrum even for massless modes.

5. We find that the new central charge Y_k associated with the junction gives a negative contribution to the mass of the domain wall junction, whereas the central charge Z_k gives a dominant positive contribution. One has to be cautious to identify the central charge Y_k alone as the mass of the junction.

REFERENCES

1. H. Oda, K. Ito, M. Naganuma and N. Sakai, *Phys. Lett.* **B471** (1999) 148, hep-th/9910095.
2. K. Ito, M. Naganuma, H. Oda and N. Sakai, *Nucl. Phys.* **B586** (2000) 231, hep-th/0004188.
3. N. Arkani-Hamed, S Dimopoulos, and G. Dvali, *Phys. Lett.* **B429** (1998) 263, hep-ph/9803315 ; I. Antoniadis, N. Arkani-Hamed, S Dimopoulos, and G. Dvali, *Phys. Lett.* **B436** (1998) 257, hep-ph/9804398 .
4. L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 3370; *Phys. Rev. Lett.* **83** (1999) 4690 ; M. Cvetic, S. Griffies and S. Rey, *Nucl. Phys.* **B381** (1992) 301, hep-th/9201007.
5. E. Witten and D. Olive, *Phys. Lett.* **B78** (1978) 97.
6. E. R. C. Abraham and P. K. Townsend, *Nucl. Phys.* **B351** (1991) 313 ; M. Cvetic, F. Quevedo and S. Rey, *Phys. Rev. Lett.* **67** (1991) 1836.
7. G. Dvali and M. Shifman, *Phys. Lett.* **B396** (1997) 64, hep-th/9612128 ; *Nucl. Phys.* **B504** (1997) 127, hep-th/9611213 .
8. G. Gibbons and P. K. Townsend, *Phys. Rev. Lett.* **83** (1999) 1727, hep-th/9905196.
9. S. M. Carroll, S. Hellerman and M. Trodden, *Phys. Rev.* **D61** (2000) 065001, hep-th/9905217 ; D. Binosi and T. ter Veldhuis, *Phys. Lett.* **B476** (2000) 124, hep-th/9912081.
10. V.A. Rubakov and M.E. Shaposhnikov, *Phys. Lett.* **B125** (1983) 136; K. Akama, *Proc. Int. Symp.* on Gauge Theory and Gravitation, *Ed.* K. Kikkawa *et.al.* (1983) 267.
11. B. Chibisov and M. Shifman, *Phys. Rev.* **D56** (1997) 7990, hep-th/9706141 .
12. A. Kovner, M. Shifman, and A. Smilga, *Phys. Rev.* **D56** (1997) 7978, hep-th/9706089; A. Smilga and A. Veselov, *Phys. Rev. Lett.* **79** (1997) 4529, hep-th/9706217; B. de Carlos and J. M. Moreno, *Phys. Rev. Lett.* **83** (1999) 2120, hep-th/9905165;
13. V. Kaplunovsky, J. Sonnenschein, and S. Yankielowicz, *Nucl. Phys.* **B552** (1999) 209, hep-th/9811195 .
14. M. Shifman and T. tel Veldhuis, *Phys. Rev.* **D62** (2000) 065004, hep-th/9912162.
15. N. Arkani-Hamed, S Dimopoulos, G. Dvali, and N. Kaloper, *Phys. Rev. Lett.* **84** (2000) 586, hep-th/9907209; C. Csaki and Y. Shirman, *Phys. Rev.* **D61** (2000) 024008, hep-th/9908186 .